spaced thick septa. The problem is formulated so that the complexity of the evaluation procedures is not affected by the number of septa. The calculated results are in excellent agreement with other available data. Moreover, investigation of the effect of matrix truncation indicates that the CCPT solutions converge absolutely to the exact solutions, making the problem of "relative convergence" virtually nonexistent. Possible application of this approach is in the design of *E*-plane filters.

References

[1] R. Mittra and S. W. Lee, *Analytic Techniques in the Theory of Guided Waves*. New York: Macmillan, 1971.

[2] L. Lewin, *Theory of Waveguides*, New York: Halsted Press, John Wiley, 1975.

[3] J. R. Pace and R. Mittra, "The trifurcated waveguide," *Radio Sci.*, vol. 1, no. 1, pp. 117–122, Jan. 1966.

[4] A. E. Heins, "Systems of Wiener–Hopf integral equations and their application to some boundary value problems in electromagnetic theory," in *Proc. Symp. Appl. Math*, vol. 2, pp. 76–81, Math. Soc. New York, July 1948.

[5] Q. Igarashi, "Simultaneous Wiener–Hopf equations and their application to diffraction problems in electromagnetic theory," *J. Phys. Soc. Japan*, vol. 19, no. 7, pp. 1213–1221, July 1964.

[6] Y. C. Shih and T. Itoh, "*E*-plane filters with finite thickness septa," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 1009–1012, Dec. 1983.

[7] R. Safavi-Naini and R. H. MacPhie, "On solving waveguide junction scattering problems by the conservation of complex power technique," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 337–343, Apr. 1981.

[8] E. M. Sich and R. H. MacPhie, "The conservation of complex technique and *E*-plane step-diaphragm junction discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 198–201, Feb. 1982.

[9] R. Safavi-Naini and R. H. MacPhie, "Scattering at rectangular-to-rectangular waveguide junction," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 2060–2063, Nov. 1982.

[10] F. Arndt, J. Bornemann, R. Vahldieck, and D. Grauerholz, "*E*-plane integrated circuit filters with improved stopband attentuation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1391–1394, Oct. 1984.

[11] K. Chang, "Impedance calculation of three narrow strips on the transverse plane of a rectangular waveguide," *IEEE Trans. Microwave Theory Tech*, vol. MTT-32, pp. 126–130, Jan. 1984.

[12] R. Safavi-Naini, "On solving waveguide junction scattering problems by the conservation of complex power technique," Ph.D. dissertation, Univ. of Waterloo, Waterloo, Ontario, Canada, Mar. 1979.

[13] Y. C. Shih and K. G. Gray, "Convergence of numerical solutions of step-type waveguide discontinuity problems by modal analysis, in *1983 IEEE MTT-S Dig.*, pp. 233–235.

Microstrip Transmission Line With Finite-Width Dielectric and Ground Plane

CHARLES E. SMITH AND RAY-SUN CHANG

Abstract —**Design data for microstrip transmission lines with finite-width dielectric and ground plane are presented. The characteristic impedance and velocity of propagation are tabulated from results of a moment-method solution of a quasi-TEM transmission-line model of this microstrip structure.**

I. Introduction

A numerical solution for an open microstrip transmission line with a finite-width dielectric and infinite-width ground plane was recently described in a paper by Smith and Chang [1]. This case

Manuscript received March 9, 1985; revised May 2, 1985.

C. E. Smith is with the Electrical Engineering Department, University of Mississippi, University, MS 38677.

R.-S. Chang is with C. W. Trucking Service, Manvel, TX 77578.

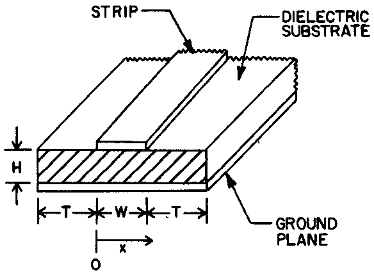

Fig. 1. Microstrip transmission line with both a truncated dielectric substrate and ground plane.

of the truncated dielectric microstrip with an infinite ground plane was considered because it more closely approximates the practical finite substrate case than the idealized infinite-width model normally employed. Consequently, a parameter study of the characteristics of this type of transmission line was presented for design purposes for practical applications.

However, another related model of some importance is that of a microstrip transmission line having both a truncated dielectric and ground plane as shown in Fig. 1. This finite-width dielectric and ground-plane structure represents several practical applications where odd-mode propagation is dominant. One such application of the structure is related to the design of tapered, balanced-to-unbalanced, transformers (baluns) such as that type originally proposed by Duncan and Minerva [2] and later used in principle by Gans, Kajfez, and Rumsey for mode conversion in microstriplines [3]. The resulting transformer employs a tapered transition which has a characteristic impedance that varies continuously in a smooth fashion from the balanced-to-unbalanced transmission line, and the cross-sectional characteristic impedance as a function of length is the desired design quantity in this approach based on the theory of small reflections [4].

A related problem consisting of two perfectly conducting zero-thickness parallel strips of unequal widths in a homogeneous medium has been analyzed to obtain an approximate solution to this class of structures for design purposes [5]. The accuracy of this data is certainly questionable because of the homogeneous modeling of this inhomogeneous structure, particularly for both small T and W/H as defined in Fig. 1. Thus, a numerical solution for the inhomogeneous configuration of Fig. 1 has been developed to obtain a better approximation of line parameters for general design purposes. Tentative results from this numerical analysis indicate that the design data for the homogeneous model is indeed in error by more than ten percent for small W/H ratios [6]. A brief discussion and the computed results of this numerical solution for the truncated dielectric and ground-plane structure are presented in the next section of this paper.

II. Numerical Solution and Results

The transmission-line characteristics for the microstrip problem of Fig. 1 can be obtained using a free-space Green's function formulation in terms of equivalent surface charge sources on the structure boundaries coupled with a moment-method solution for a quasi-TEM model. This approach has been used previously to solve inhomogeneous electrostatic problems, and the theory for this method has been presented in several forms by Smith and Chang [1], Harrington and Pontoppidan [7], Adams and Mautz [8], and Smith [9]. In addition, Rao, Sarkar, and Harrington have recently used this same surface charge formulation to analyze electrostatic fields of conducting bodies in multiple dielectric

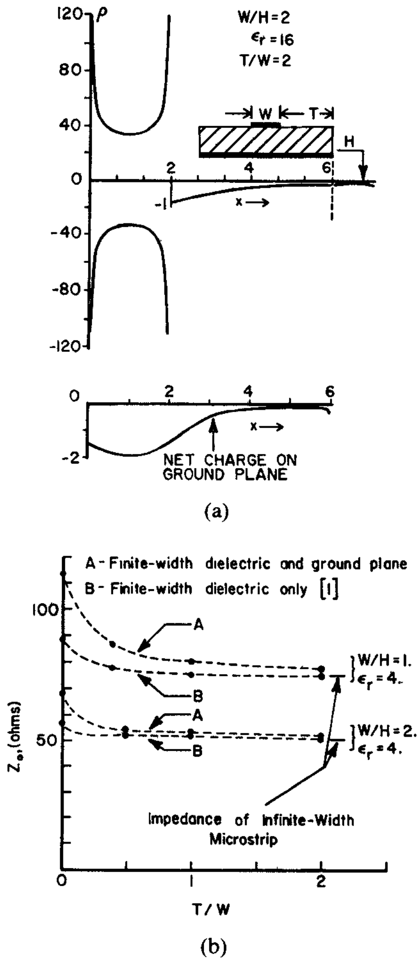

Fig. 2. (a) Charge distribution on the strip, ground plane, and dielectric interface boundaries ($W/H = 2$, $\epsilon_r = 16$ and $T/W = 2$). Vertical axis is relative charge-density. (b) Characteristic impendance versus T/W for similar microstrip transmission lines with A: finite-width dielectric and ground plane and B: finite-width dielectric only (see [1] for data).

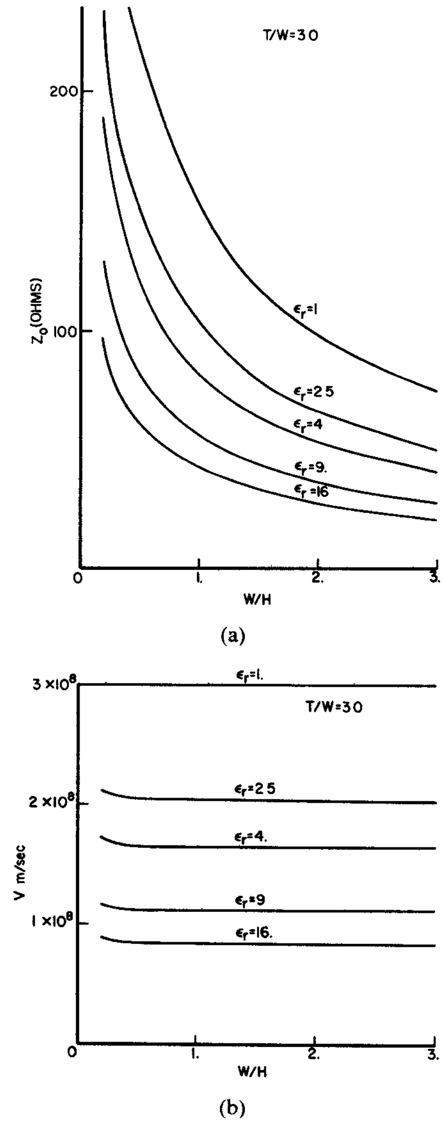

Fig. 3. (a) Characteristic impedance of microstrip for $T/W = 3.0$. (b) Velocity of propagation on microstrip for $T/W = 3.0$.

media [10]. Mathematical descriptions of this surface charge approach showing details can be found in [1], [7]–[10].

As mentioned earlier, Smith and Chang [1] have extended the basic technique to the finite-width dielectric microstripline through numerical analysis in conjunction with a brief experimental investigation. In the investigation presented herein, the numerical analysis for the earlier finite-width dielectric microstrip has been modified to include modeling with a truncated ground plane, as well as the dielectric. The defining equations of [1] remain the same for this new problem, and the modification involved changes in the numerical formulation only because image theory cannot be employed.

A modified computer program based on this previous formulation has been created to compute the equivalent surface charge densities for the boundary value problem for the odd-mode model of the microstrip of Fig. 1. Because the application of the method of images cannot be used in this case, the conductors of Fig. 1 are treated as two symmetrical lines of unequal widths in an inhomogeneous media.

From the associated computations, examples of plots of the charge distributions on the strip, ground plane, and dielectric interface boundaries are presented in Fig. 2(a) for physical parameters where $W/H = 2$, $\epsilon_r = 16$, and $T/W = 2$, and plots of the characteristic impedance as a function of the truncation ratio T/W are given in Fig. 2(b). A comparison of these charge distribution results with a related case by Smith and Chang for the infinite ground-plane truncated substrate microstrip [1] shows that the finite-width ground plane does influence the equivalent surface charge density near the ground-plane edges, as well as the relative magnitude of the charge on the strip. This result is expected because the truncation of the ground plane changes the resultant field structure and, thus, the transmission-line parameters.

The computed odd-mode characteristic impedance and velocity of propagation as a function of W/H ratios have been computed and are presented in Figs. 3–8 for several values of truncation (T/W). For design purposes, it is well to note that, for $T/W > 2$ and $W/H > 1$, these transmission line parameters are near that of the infinite substrate case (see [9] or [11]) as shown in Fig. 2(b).

The numerical results for Z_0 as a function of T/W of Fig. 2(b) indicate the effect of the truncated ground plane on the characteristic impedance of the microstripline as contrasted with the characteristic impedance of the finite-width dielectric case [1]. The characteristic impedance of the line with finite-width dielectric and ground plane appears to be approaching a higher value than that of the finite-width dielectric for decreasing T/W. This trend results from the fact that, in the case of the finite-width

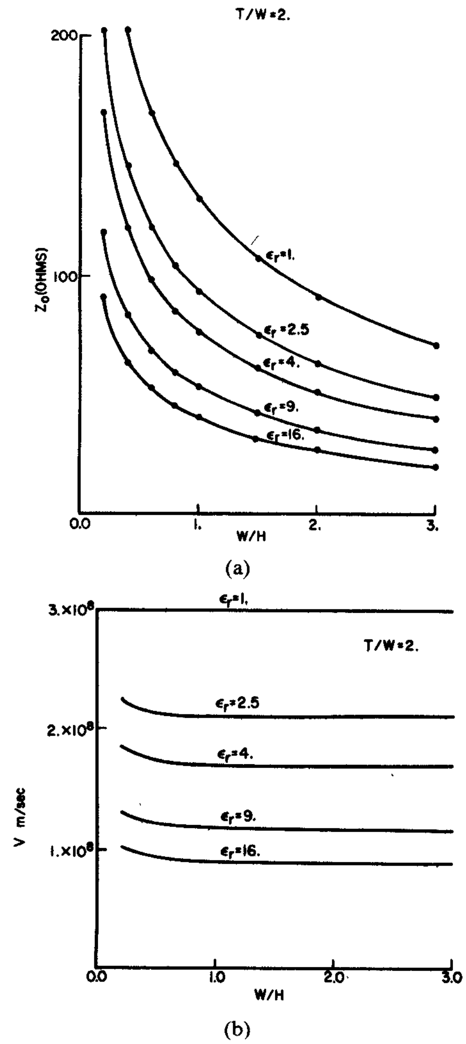

Fig. 4. (a) Characteristic impedance of microstrip for $T/W = 2.0$. (b) Velocity of propagation on microstrip for $T/W = 2.0$.

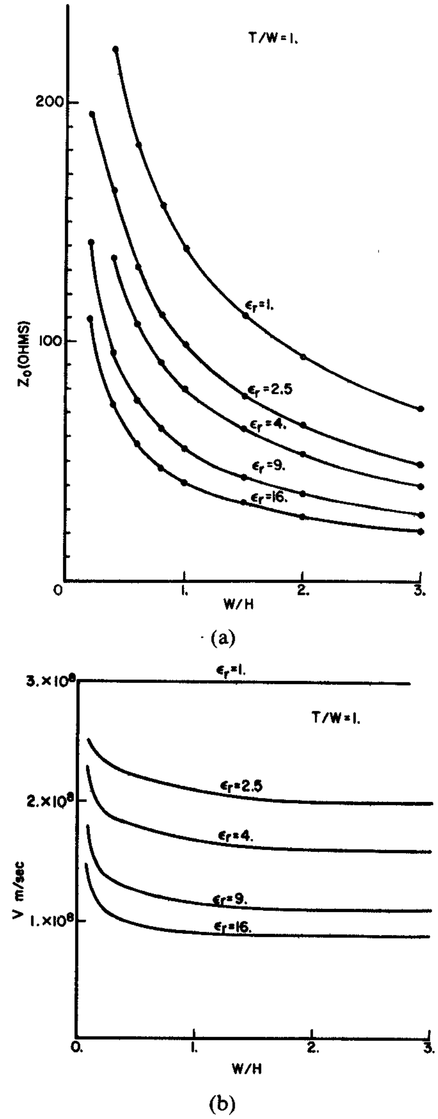

Fig. 5. (a) Characteristic impedance of microstrip for $T/W = 1.0$. (b) Velocity of propagation on microstrip for $T/W = 1.0$.

ground plane, the configuration is approaching a two-strip geometry with spacing H for $T/W \to 0$, where the finite-width dielectric geometry of [1] approaches a two-strip geometry with spacing of $2H$ for $T/W \to 0$ based on image theory.

III. Design Example Using Finite-Width Ground-Plane Data

As a demonstration of the validity of the design data presented for a microstripline with both a truncated dielectric and a ground plane for odd-mode propagation, a 50–73-Ω tapered balun was designed and constructed as shown in Fig. 9. In this balun, a linear impedance taper was selected for a 3.8-cm section of line although any taper could be selected in practice. The substrate material had a relative dielectric constant of 2.2 and a thickness of 0.62 in as described in the insert of the accompanying figure.

The actual impedance level at any given position on the truncated microstrip is, of course, directly dependent on the combination of both W/H and T/W ratios. Thus, there exist many choices for design strategies based primarily on strip width and truncation. For simplicity and ease of design, an arbitrary taper in T/W of 3, 2, 1, 0.5, 0.25, and 0 for equally spaced points was selected for testing and measurement purposes.

In the test design, the balanced-to-unbalanced tapered line was started from a microstrip transmission line with $T/W > 5$, a value which approaches that of the infinite-width dielectric case. In order to maintain an impedance match between the input 50-Ω microstrip and the truncated taper line, a small step change in actual line width was required as indicated by the W/H ratios in Table I and as can be seen in Fig. 9. To begin the design, the 50-Ω input W/H ratio was determined from design data presented in [1] and [10], and, next, the 73-Ω output W/H was determined from information presented in Fig. 8 for $T/W = 0$. With the input and output W/H ratios for the selected laminate, along with the specified impedance and T/W tapers, the line parameters as a function of position for the six equally spaced points were determined from Figs. 3–8. These parameters are given in Table I.

Plots of the measured and computed reflection coefficient as a function of position are given in Fig. 10 for a first attempt of a prototype balun. A comparison of this data indicates good agreement in this limited test with a small difference in the impedance transformation (approximately 2 percent). The very close agreement of the measured and computed reflection coefficients indicates that the performance of the balun in an odd-mode impedance transformation is valid and that very small asymmetry of unbalanced excitation currents exists on the lines. It further

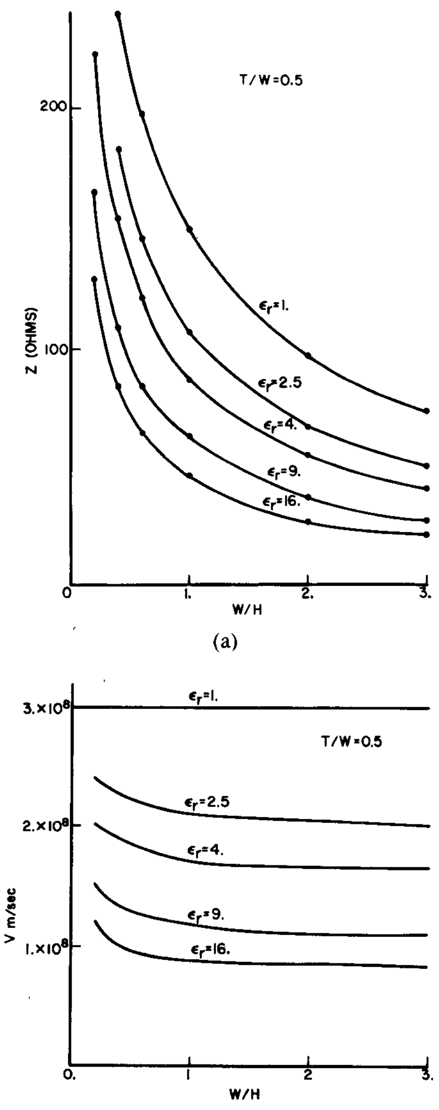

Fig. 6. (a) Characteristic impedance of microstrip for $T/W = 0.5$. (b) Velocity of propagation on microstrip for $T/W = 0.5$.

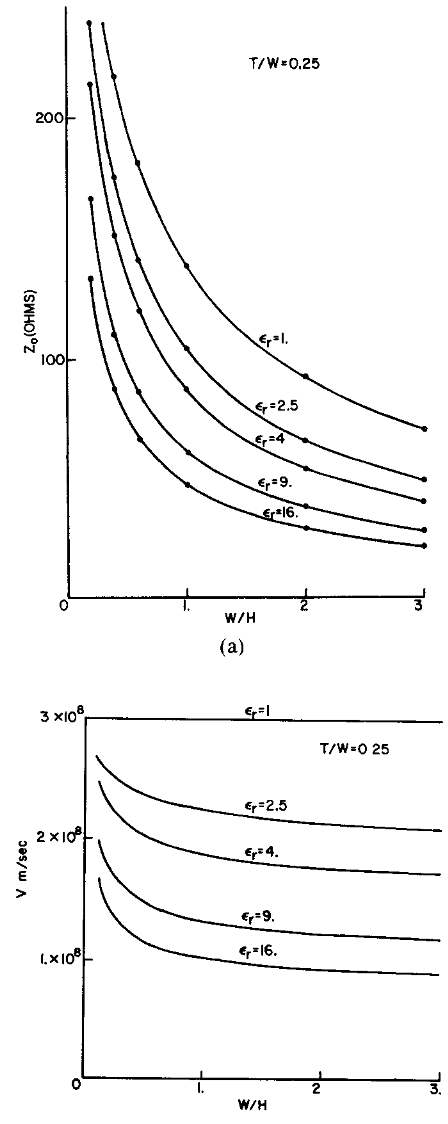

Fig. 7. (a) Characteristic impedance of microstrip for $T/W = 0.25$. (b) Velocity of propagation on microstrip for $T/W = 0.25$.

supports the validity of the use of this numerical procedure to compute line parameters for the quasi-TEM propagation mode for a microstrip transmission line with both truncated dielectric and ground plane. Thus, the curves of Figs. 3–8 should be useful for design purposes in practical applications where odd-mode operation of microstrip with truncated dielectric and ground plane is required.

IV. Summary

Numerical results of a moment-method solution for a microstrip transmission line with finite-width dielectric and ground plane are presented in this paper for several different truncation-to-strip width ratios (T/W). These solutions were obtained for a surface equivalent model of the related inhomogeneous boundary value problem using pulse expansion terms and point matching in a moment analysis. For large W/H ratios, these results appeared to converge satisfactorily using up to 200 total subsection expansion terms on the basic model as depicted in Figs. 1 and 2(a). For very large W/H ratios, the results approached the published data for the infinite-width microstriplines as given in the referenced papers within a small percent difference. Improvement in the convergence for very small W/H ratios ($W/H \ll 1$) could probably be made over that obtained in this investigation where the use of equal, uniform-width expansion terms forced the available computer memory size to limit the numerical analysis. However, overall experience, including the previous test example, indicates that the graphical data of Figs. 3–8 are quite accurate for engineering design purposes and for ascertaining the effect of truncating the dielectric and ground plane of a microstripline.

The results obtained in this numerical analysis as shown in Figs. 2(b)–8 indicate that the truncation of the dielectric and ground plane has a noticeable effect for truncation ratios $T/W < 2$. As would be expected, this effect appears to be more pronounced as the width-to-height ratio (W/H) and/or the relative dielectric constant (ϵ_r) are decreased. In microstriplines, the fields are strongest underneath the strip for large W/H and/or ϵ_r. As W/H and/or ϵ_r are decreased, the stored energy or field strength outside the strip region increases which causes the exterior boundaries, i.e., the finite-width ground plane and substrate, to have more effect on the line characteristics.

This transmission-line data for the finite-width dielectric and ground plane can thus be used to predict the effect of ground-plane truncation on transmission characteristics for microstrip circuit design and applications. The design example given further

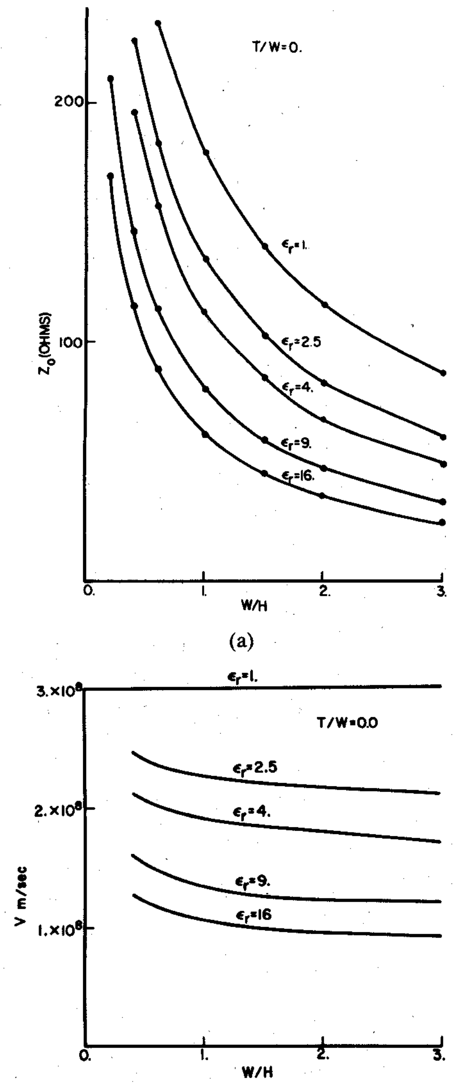

Fig. 8. (a) Characteristic impedance of microstrip for $T/W = 0$. (b) Velocity of propagation on microstrip for $T/W = 0$.

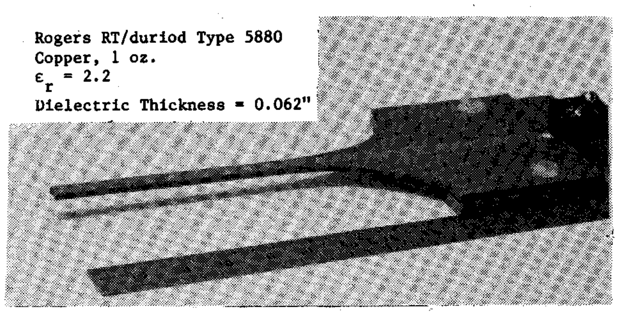

Fig. 9. Physical layout of 50–73-Ω balun with laminate characteristics.

supports the use of this data for design where the physical environment demands that such a modified microstrip geometry be employed.

References

[1] C. E. Smith and R. S. Chang, "Microstrip transmission line with finite-width dielectric," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 90–94, Feb. 1980.

[2] J. W. Duncan and V. P. Minerva, "100:0 bandwidth balun transformer," *Proc. IRE*, vol. 48, pp. 156–164, Feb. 1960.

[3] M. Gans, D. Kajfez, and V. H. Rumsey, "Frequency independent baluns," *Proc. IEEE*, vol. 53, pp. 647–648, June 1965.

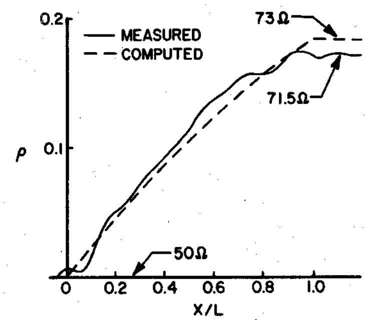

Fig. 10. Measured and computed reflection coefficients for linear impedance taper balun (50–73 Ω). Measurements made using time-domain reflectometer (TDR) with 28-ps risetime and 3-percent magnitude accuracy.

TABLE I
LINE PARAMETERS FOR 50–73-Ω MICROSTRIP BALUN

Normalized Position X/L	Z_0, Ω	T/W	W/H*
<0.0	50.	>5.	3.12
0.0	50.	3.	3.31
0.2	54.6	2.	2.91
0.4	59.2	1.	2.68
0.6	63.8	0.5	2.33
0.8	68.4	0.25	2.27
1.0	73.	0.0	2.53
>1.0	73.	0.0	2.53

*Graphical design data used to determine basic design. Final W/H checked with computer results which are presented in table.

[4] R. E. Collin, *Foundations for Microwave Engineering*. New York: McGraw-Hill, 1966, ch. 5.

[5] R. K. L. Poon, "Capacitance of a microstrip of unequal widths in a homogeneous medium," *Electron. Lett.*, vol. 15, no. 2, pp. 44–45, Jan. 1979.

[6] C. E. Smith, "On the accuracy of design data for microstrip of unequal widths in an inhomogeneous media," *Electron. Lett.*, vol. 19, no. 15, pp. 575–576, July 21, 1983.

[7] R. F. Harrington and K. Pontoppidan, "Computation of Laplacian potentials by an equivalent source method," *Proc. Inst. Elec. Eng.*, vol. 116, no. 10, pp. 1715–1720, Oct. 1969.

[8] A. T. Adams and J. R. Mautz, "Computer solution of electrostatic problems by matrix inversion," in *Proc. Nat. Electronic Conf.*, vol. 25, Dec. 1969, pp. 198–201.

[9] C. E. Smith, "A coupled integral equation solution for microstrip transmission lines," *IEEE G-MTT Microwave Symp. Dig.*, June 1973, pp. 284–286.

[10] S. M. Rao, T. K. Sarkar, and R. F. Harrington, "The electrostatic field of conducting bodies in multiple dielectric media," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1441–1448, Nov. 1984.

[11] T. G. Bryant and J. A. Weiss, "Parameters of microstrip transmission lines and of coupled pairs of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 1021–1027, Dec. 1968.

Correction to "Waveguide Modes Via an Integral Equation Leading to a Linear Matrix Eigenvalue Problem"

G. CONCIAURO, M. BRESSAN, AND C. ZUFFADA

After having examined the above paper,[1] we noticed that the symbol of principal value in integral (1) is inappropriate. In fact, integral (1) can represent the field on σ only in the limit as the observation point approaches σ. This oversight, however, does not affect the theory at all.

Furthermore, in (5), the symbol δ' should be read as ∇'.

Manuscript received May 6, 1985.
The authors are with the Dipartimento di Elettronica dell'Università di Pavia, 27100 Pavia, Italy.
[1] G. Conciauro *et al.*, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1495–1504, Nov. 1984.